MATH REVIEW #2 SUPPLEMENT (MATH CAMP 2007)

JOHN MORROW

What follows is a rough sketch of material for Day 2 and a few more problems.

1. Day 2

(1) Functions
  (a) Example: $\mathbb{R}^1 \to \mathbb{R}^1$
  (b) Monotone functions.
  (c) **Problem:** Construct examples on $[0, 1]$ of the following types of functions and find the points in $[0, 1]$ where they are maximized and minimized:
    (i) Strictly increasing.
    (ii) Strictly decreasing.
    (iii) Increasing but not strictly increasing.
    (iv) A function which fails both definitions of increasing and decreasing functions.
  (d) Example: $\mathbb{R}^2 \to \mathbb{R}^2$
  (e) Example: $\mathbb{R}^N \to \mathbb{R}^1$
  (f) Concave/Convex functions.
(2) Smoothness
  (a) Sequences.
  (b) Open/closed intervals.
  (c) Closed and bounded intervals, convergence of sequences.
  (d) **Problem:** Suppose for a given sequence $\{a_n\}$ that $|a_n| \leq M$ for some constant M. Show that $\{a_n\}$ has a convergent subsequence. Does

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$\{a_n\}$ have to converge to a unique number a? (Prove or provide a counterexample.)

(e) Continuity.

(f) Continuous functions on closed and bounded sets always take on maximal and minimal values.

(g) **Problem:** $f(x) \equiv 1/|x|$ fails to take on a maximal value. How does this fail to happen?

(h) The derivative on $\mathbb{R}^1$.

(i) **Problem:** Show that continuity of $f(x)$ at y is a necessary condition for f to be differentiable at y.

(j) $\mathcal{C}^1$, $\mathcal{C}^2$, $\mathcal{C}^\infty$.

(3) Optimization Primer

(a) Local/Global Maxima and Minima.

(b) Uniqueness of maxima/minima for strictly concave/convex functions.

(c) **Problem:** Provide an example of a function which is concave and convex.

(d) Necessity of critical points for maxima/minima.

(e) **Problem:** Construction a function $f(x)$ where for some y we have $f'(y) = 0$ but $f'(y)$ is not a local max or min.

(f) Critical points sufficient for maxima/minima under concavity/convexity.