MATH REVIEW #3 SUPPLEMENT (MATH CAMP 2007)

JOHN MORROW

What follows is a rough sketch of material for Day 3.

1. Day 3

(1) Optimization Primer
   (a) Local/Global Maxima and Minima.
   (b) Uniqueness of maxima/minima for strictly concave/convex functions (Mention Multivariate Case).
   (c) Necessity of critical points for maxima/minima.
   (d) Critical points sufficient for maxima/minima under concavity/convexity.
   (e) Basic Checklist for simple problems.
      (i) Check continuity.
      (ii) "Compactify" Problem.
      (iii) Check for concavity/convexity.
      (iv) Find critical points.
      (v) Be creative and use your intuition!
   (f) Examples: Monopolist Profit Maximization
      (i) Linear Demand, constant costs:

$$P(q) \equiv a - bq \;\; C(q) \equiv c$$

      (ii) Linear Demand, constant marginal costs:

$$P(q) \equiv a - bq \;\; C(q) \equiv cq$$

Date: 8/26/07.

(iii) Fancy Demand, constant marginal costs:

$$P(q) \equiv a - bq^{\alpha} \quad C(q) \equiv cq$$

(g) Examples: Cost minimization

(i) Need to produce q units, have two technologies:

$$c_1(q) \equiv cq \quad c_2(q) \equiv q^{\frac{1}{2}} + k$$

(2) Some Miscellaneous Items

(a) Exp and Log properties, change of base.

(b) Elasticity, $\mathcal{E}_{q,p}$ def, as %change and as log deriv.

(c) Homogeneous functions.

(3) More Derivatives

(a) Partial derivative notations, dx and $Df(x)$.

(b) Derivative as a linear operator

$$f(x) = \sum_{i=1}^{N} i \cdot x^i \quad f(x) = \sum_{i=1}^{N} e^{i\beta x}$$

(c) Partial derivatives

$$f(x,y) = x^{\alpha}y^{\beta} \quad f(x_1, \ldots, x_N) = (\sum \alpha_i x_i^{\rho})^{\frac{1}{\rho}}$$

(d) Definition of derivative for $\mathbb{R}^N \to \mathbb{R}$, how it looks, Hessian, "Young's Theorem".

(e) Hessian and Strict Concavity/Convexity.

(f) Sufficient conditions for multivariate maxima.

(4) More Linear algebra (doubt we'll make it this far today).

(a) Dot product, vector "multiplication" and distance.

(b) Solving $Ax = b$, inverses.

(c) Eigenvectors/Eigenvalues and Definiteness.