Day 3 (Again, less important questions are marked with ***.)

(1) From Rebecca's problems: 6 (only answer for continuity), 8, 16 (only answer whether a matrix is negative definite/positive definite/or neither).

(2) Classify whether the following sets are open, closed, or neither:
   (a) $\mathbb{R}\backslash(0,1)$
   (b) $\mathbb{R}\backslash[0,1]$
   (c) ***$(0,1)\bigcup[2,3]$
   (d) ***$A\bigcup B$ where A and B are closed
   (e) ***$\mathbb{R}$
   (f) ***$\emptyset$

(3) Does a convex set have to be open or closed? Can a convex set be neither? (Draw examples.)

(4) Determine whether the following sequences are convergent or not, and if a sequence is convergent, write down the limit.
   (a) $a_n = \frac{1}{n}$
   (b) $a_n = \sum_{i=0}^{n} (\frac{1}{2})^i$
   (c) $a_n = \frac{n-1}{n}$
   (d) $a_n = n$
   (e) $a_n = (2)^{\frac{1}{n}}$
   (f) $a_n = a_{n-1} - 1$, $a_0 = 0$
   (g) ***$a_n = \frac{a_{n-1}+a_{n-2}}{2}$, $a_0 = 1, a_1 = 2$
   (h) ***$a_n = \sum_{i=0}^{n} \frac{1}{2(i+1)}$

(5) Determine whether the following matrices are positive definite, negative definite, or neither:
   (a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
   (b) $\begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & -9 \end{bmatrix}$
   (c) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
   (d) *** $A^\top A$ where A is a square matrix. (Hint: Check all the definitions.) What about $AA^\top$?