

Day 3 (Again, less important questions are marked with \*\*\*)

- (1) From Rebecca's problems: 6 (only answer for continuity), 8, 16 (only answer whether a matrix is negative definite/positive definite/or neither). (See answers online).
- (2) Classify whether the following sets are open, closed, or neither:
  - (a)  $\mathbb{R} \setminus (0, 1)$  (Closed)
  - (b)  $\mathbb{R} \setminus [0, 1]$  (Open)
  - (c)  $*** (0, 1) \cup [2, 3]$  (Neither)
  - (d)  $*** A \cup B$  where  $A$  and  $B$  are closed (Closed)
  - (e)  $*** \mathbb{R}$  (Open and Closed)
  - (f)  $*** \emptyset$  (Open and Closed)
- (3) Does a convex set have to be open or closed?  
 No, take  $[0, 1)$   
 Can a convex set be neither? (Draw examples.)  
 Same example works.
- (4) Determine whether the following sequences are convergent or not, and if a sequence is convergent, write down the limit.
  - (a)  $a_n = \frac{1}{n}$   
 $a_n \rightarrow 0$
  - (b)  $a_n = \sum_{i=0}^n (\frac{1}{2})^i$   
 $a_n \rightarrow 2$
  - (c)  $a_n = \frac{n-1}{n}$   
 $a_n \rightarrow 1$
  - (d)  $a_n = n$   
 Not convergent.
  - (e)  $a_n = (2)^{\frac{1}{n}}$   
 $a_n \rightarrow 1$
  - (f)  $a_n = a_{n-1} - 1, a_0 = 0$   
 Not convergent.
  - (g)  $*** a_n = \frac{a_{n-1} + a_{n-2}}{2}, a_0 = 1, a_1 = 2$   
 $a_n \rightarrow 1\frac{2}{3}$  (This was pretty tricky! Good for you if you got it.)
  - (h)  $*** a_n = \sum_{i=0}^n \frac{1}{2^{(i+1)}}$   
 Not convergent.
- (5) Determine whether the following matrices are positive definite, negative definite, or neither:
  - (a)  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$   
 Neither
  - (b)  $\begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & -9 \end{bmatrix}$   
 Neither
  - (c)  $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$   
 Neither
  - (d)  $*** A^T A$  where  $A$  is a square matrix. (Hint: Check all the definitions.) What about  $AA^T$ ?  
 Both are positive definite iff  $A$  is invertible.