

(1) Day 1 Partial Solutions

- (a) Day 1 Problem (Below) (Ask me in person.)
 (b) Problems 1,2 (for 2 you can draw Venn Diagrams if you like), 4,5,7. (See online answers.)
 (c) Fill in the following logic table:

To make things easier I will first fill in $\neg A$

A	B	$\neg A$	$(\neg A) \implies B$	$(\neg A) \wedge B$	$A \iff B$
T	T	F	T	F	T
T	F	F	T	F	F
F	T	T	T	T	F
F	F	T	F	F	T

- (d) For what values of α and x are the following functions Strictly/Weakly: Concave? Convex? Increasing? Decreasing? (These functional forms will appear over and over again.)

$f(x)$	$f'(x)$	Strictly Increasing	Strictly Decreasing
x^α	$\alpha x^{\alpha-1}$	$\alpha > 0, x > 0$	$\alpha < 0, x > 0$
$\alpha \ln(x)$	$\frac{\alpha}{x}$	$\alpha > 0, x > 0$ and $\alpha < 0, x < 0$	$\alpha < 0, x > 0$ and $\alpha < 0, x > 0$
$e^{\alpha x}$	$\alpha e^{\alpha x}$	$\alpha > 0, x \in \mathbb{R}$	$\alpha < 0, x \in \mathbb{R}$
αx	α	$\alpha > 0, x \in \mathbb{R}$	$\alpha < 0, x \in \mathbb{R}$

$f(x)$	$f''(x)$	Strictly Concave	Strictly Convex
x^α	$\alpha(\alpha-1)x^{\alpha-2}$	$\alpha^2 < \alpha, x > 0$	$\alpha^2 > \alpha, x > 0$
$\alpha \ln(x)$	$-\frac{\alpha}{x^2}$	$\alpha > 0$	$\alpha < 0$
$e^{\alpha x}$	$\alpha^2 e^{\alpha x}$	—	$\alpha, x \in \mathbb{R}$
αx	0	—	—

- (e) In general, for $\alpha > 0$, the following formula holds:

$$\sum_{i=0}^N \alpha^i = \frac{1 - \alpha^{N+1}}{1 - \alpha}$$

Sometimes the following formula is also true:

$$\sum_{i=0}^{\infty} \alpha^i = \frac{1}{1 - \alpha}$$

For which values of α should the second equation hold? Relate the second equation to the first. The formula holds so long as $\alpha \in (-1, 0) \cup (0, 1)$. (For $\alpha = 0$ this depends on how you define 0^0 , so answers may vary.) For $\alpha \in (-1, 0) \cup (0, 1)$ as N gets large, α^{N+1} goes to 0 so the first equation becomes the second.

- (f) ***The *power set* of a set A , denoted $P(A)$ is defined as the collection of all subsets of A , or rather

$$P(A) \equiv \{B : B \subset A\}$$

Find $P(A)$ for $A = \{1, 2\}$ and $A = \{\{1\}, 2, \{3\}\}$. If A contains N elements, how many elements does $P(A)$ have?

(Ask me in person if you are interested.)

- (g) Which of the following sets are convex?

- (i) $[0, 1]$ (Convex)
 (ii) $(10, 100)$ (Convex)
 (iii) $(0, 1) \cup (20, 100)$ (Not Convex)
 (iv) $(1, 2) \cup (2, 3)$ (Not Convex)
 (v) *** $A \cap B$ where A and B are convex (Convex, ask me in person).