Endowments, Equality, and Power Relationships

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Motivation

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The Question:
Can we explore such behavior where the consequences are *endowment driven*. In other words, can the wealthy/strong exploit the poor/weak in a general setting? Can we uncover *power relationships* in such an abstract setting?

Narrative Example

- Map the model features onto the recent conflict in Iraq.
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- In a nutshell, an Iraq application of this work is "Applying Cournot strategy concepts to conflicts arising from institutional change."
- Gives us a formal way to talk about conflict ensuing from a radical change in institutions in Iraq.

Modelling Power

There is a huge literature and much debate on how to define *power* and how to model it. We will commit the usual sin of the economist and posit an unrealistic, naive and oversimplified model. Hopefully more realistic modifications can be made while maintaining a setting which avoids the criticism of injecting one's conclusion into assumptions.

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Strengths of this particular approach:

1. Ties into existing literature and is sufficiently general to include Cournot competition.
2. Characterization of behaviors based on elasticities and observable parameters which are suited to empirical work.
3. Tractable description of conflict with > 2 agents (literature is almost always two agents.)

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Actions:

Each agent i decides to split R_i between *effort* e_i devoted to joint (public, national) production and *argument* a_i which influences what share of joint production the agent receives. We denote the aggregate level of argument A where $A = \sum_{i=1}^{N} a_i$.

Actors, Actions and Production

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Joint Production:

Determined by a strictly increasing, strictly concave function f which pools all effort levels. Thus total production is given by

$$f(\sum_{i=1} e_i) \quad = \quad f(R - A)$$

Payoffs:

Given a level of joint production $f(R-A)$, payoffs i are determined by the share of production received $p_i(a)$. p is a technology that "converts" resources into control over the fruits of production. We thus term it a *power function.* Payoffs are therefore defined as

Definition (Payoffs)

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Payoffs and Conflict Equilibrium

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Conflict Equilibrium:
The equilibrium concept is simply that each level of argument a_i is a best response to the vector of other groups' arguments a_{-i}. In other words, a Cournot Equilibrium. $N-$firm Cournot competition of monopolists fits within this framework (we'll get to that).

"Power Technologies"

Following the literature on contests[1], we restrict p to be of the form (for some h):

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- h determines how economic resources a_i can be "mobilized" into conflictual resources $h(a_i)$.
- $p_i(a)$ is the share of joint production that group i receives: it is precisely the share of "conflictual resources" $h(a_i)$ a group has relative to total conflictual resources.

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An Example: Cournot Competition

The Canonical Form:
Consider N firms who choose production quantities q_i at constant marginal cost c. Each agent also faces a capacity constraint R_i so that $q_i \leq R_i$. As an industry the firms produce $Q = \sum q_i$ and jointly face inverse demand D of

$$D(Q) = g - hQ$$

so that industry profits are given by

$$\Pi(Q) = [D(Q) - c]Q = (g - c)Q - hQ^2$$

Cournot Competition as Conflict

- Firms "fight" each other by choosing quantities q_i which in aggregate will exceed the optimal quantity of a single monopolist ("dictator") . This *decreases joint profits* in the pursuit of a *larger share* of industry profits $\Pi(Q)$.

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- We can recast industry profits as a "joint production" function f as

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- Individual payoffs are given by

$$\pi_i(q_i, q_{-i}) = \frac{q_i}{Q}\Pi(Q) = \frac{h(q_i)}{\sum h(q_j)} f(R - Q)$$

Implications

- Agents must be concerned with both increasing production and increasing their share of production. Those with a greater "stake" have a greater vested interest in joint production.

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- Agents must be concerned with both increasing production and increasing their share of production. Those with a greater "stake" have a greater vested interest in joint production.
- We should expect high endowment agents to come out ahead of low endowment agents. Since production is joint, this implies high endowment agents receive a larger *share* of production.
- Behavior is determined by the relationship of the *power* function to the *production* function.
  - We might think of equitible institutions as specifications for power which preclude "strong" agents from capturing an inordinate share of joint production through the power function.
  - Structural changes in the power function will induce radically different incentives and equilibria: e.g. democratization of Iraq.

Functional Forms for Power

Relationships between production and power may be more easily characterized by elasticities.

Definition (Elasticities)

$$\eta_i^h \equiv \frac{h'(a_i)a_i}{h(a_i)} \text{ (Power)} \qquad \eta_i^f \equiv \frac{f'(R-A)a_i}{f(R-A)} \text{ (Production)}$$

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Power can range from "high" exponential returns to "low" logarithmic returns. We can incorporate a range of institutions which vary in the capability of agents to "exploit" one another.

Examples for power functions

Label	Form	Relevant Property	η_i^h
"Difference"	$h_d(x) = \alpha e^{kx}$	$p(x+\lambda) = p(x)$	kx
"Scaling"	$h_s(x) = \alpha x^{\beta}$	$p(\lambda x) = p(x)$	β
"Logarithmic"	$h_e(x) = \alpha (\ln x)^c$	$p(x_1^{\lambda}, \ldots, x_N^{\lambda}) = p(x)$	$c / \ln x$

Regularity Conditions

For formal results we need a little more structure. To this end we have the following regarding the choice of power technology:

Definition (Regularity)

$$a_i \cdot (\eta_i^h)' \leq \eta_i^h$$

Essentially this condition guarantees log concavity for each agent's decision problem. Is is generally stronger than needed, but is easy to verify for every example for power functions in the table above.

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Theorem (Existence)

Assume regularity. Then at least one equilibrium a^* exists. In addition, if $h'' < 0$ then a^* is unique.

For brevity, we will hereafter assume regularity for all results.

Applications

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- Resource endowments for conflict (R_i). E.g. Group size, wealth, % government positions held, political positions held, geographic control over strategic resources, militia strength.
- Institutional variables (h parameters). E.g. political rights, media freedom, martial oppression.

Power Relationships

We now have a general model where differences in outcomes can only arise from differences in capacities to argue and produce. In other words, the rules of the game are the same for each group and each group acts only in its immediate interest.

Can we uncover *power relationships* in this setting? I say yes!

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- (Classification) Agents are naturally partitioned into two groups, strong "Haves" and weak "Have Nots".
- (Behavior) When one agent "attacks" all the others "retreat" (Iraq Troop Surge?).
- (Outcomes) When conflict occurs and the smoke clears, redistribution towards equality *always* occurs.

Theorem (Haves and Have Nots)

Assume that $h'' < 0$. Then in the unique equilibrium a^*, agents may be partitioned into two groups, the "haves" $\mathcal{H}(a^*)$ and "have nots" $\mathcal{HN}(a^*)$ where

- *Every member of $\mathcal{H}(a^*)$ receives the same share of production, and this share is larger than that of any member of $\mathcal{HN}(a^*)$.*
- *Every member of $\mathcal{H}(a^*)$ has a strictly higher endowment than any member of $\mathcal{HN}(a^*)$.*

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The result says there are always two groups in equilibrium:

- A Strong/Wealthy group ($\mathcal{H}(a^*)$) with the same levels of argument and receive the same shares, almost as if they were "colluding."
- A Weak/Poor group ($\mathcal{HN}(a^*)$) who are heterogeneous in argument levels and shares. These agents would like to argue more but are constrained by their endowments.

If one agent "becomes aggressive" by increasing her level of argument, do the other agents respond by "fighting" with increased argument or "flee" by reducing their argument? We provide an answer with:

Theorem (Flight)

Let $a_i^*(a_{-i})$ denote agent i's optimal response to a_{-i}. Then $a_i^*(a_{-i})$ is weakly decreasing in a_{-i}.

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- In other words, agents back down in response to aggression.
- When an agent backs down, he redirects resources that would have been allocated to argument into joint production. Thus an aggresive agent's share increases and other agents become "subservient" by redirecting resources into production.

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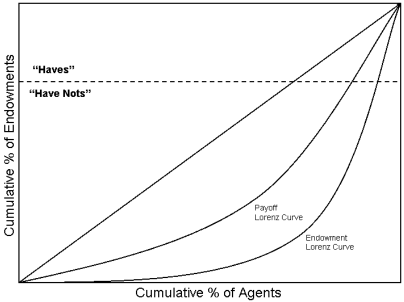

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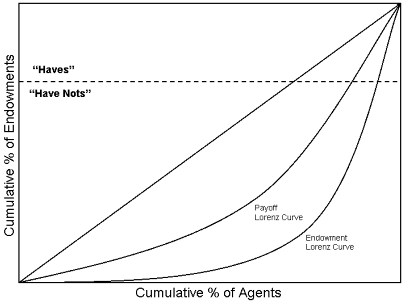

Finally, we highlight a result which relates a “resource shock” to joint production.

Suppose some level K of aid successfully reaches “productive use” in a nation receiving aid, say in a civil war. In other words:

Resource Shock

$$\pi(a_i, a_{-i}) = p_i(a)f(K + R - A)$$

This can have unintended effects with regard to increasing conflict over these new resources.

Theorem (The rich get richer...)

Let $K > 0$ and denote the pre- and post- aid argument levels by a and a'. If $\mathcal{HN}(a) \neq \emptyset$ then:

- *(Post-Aid Argument increases)* $a' > a$
- *(Post-Aid Production increases)* $f(K + R - A') > f(R - A)$
- *(Post-Aid "Haves" receive larger shares)*

$$i \in \mathcal{H}(a') \implies p_i(a') > p_i(a)$$

- *(Pre-Aid "Have nots" receive smaller shares)*

$$i \in \mathcal{HN}(a) \implies p_i(a') < p_i(a)$$

In particular, (1) shows that more resources are devoted to non-productive activity but (2) shows this does not eliminate the productive effect of K. (3) and (4) taken together show that inequality is increasing.

Directions (Please Suggest!)

- Minimal Shares.

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Conclusion

Thank you!