ADDITIONAL MATERIALS

What follows are materials that would usually be under the heading of "available upon request." Specifically, additional appendices with examples, proofs, and documented source code for the simulations used in estimation.

1. Appendix I: Data Manipulation

We are interested in MSM who exclusively and actively have sexual relationships with men. Accordingly we dropped all female subjects and those answering anything but "Only Men" or "Mostly Men" to a question regarding gender preference. Two remaining individuals were dropped who had exclusively female partners. Another individual who answered "Don't Know" to his own attractiveness rating was dropped, as this is a primary variable of interest. Finally, two individuals who had fewer than two partners in the last five years were dropped.

As this is survey data, we dropped three other observations based on population characteristics before analysis. We acknowledge them here specifically due to the subjective nature of removal. Two individuals who answered that homosexual sex was "always wrong" to a sex attitude question were dropped both for the content of the question and because they answered 1 on the attitude scale from 1 to 4 with no 2's recorded in the sample. Another individual who uniquely reported himself as the most unattractive member of the population claimed about 10% of the most attractive sexual partners in the entire sample. In fact, unless the sex market is perfectly assortative, the model predicts such behavior, but given that there is only one individual reporting this level of attractiveness we have removed him from the data.

The variable for frequency of sex over the last year must be reconstructed as pointed out in the data reference, using sex frequency for those with only one partner found in P1OFTEN and otherwise using SEXFREQ. Unfortunately the underlying variables don't exactly mesh but we construct sex frequency as follows:

Table 1. Sex Frequency Construction.

P1OFTEN		SEXFREQ	P1OFTEN		SEXFREQ
"Once a day or more"	$\longrightarrow$	1+ times a day	"2 to 3 times a month"	$\longrightarrow$	2-3 times a month
"4 to 6 times a week"	$\longrightarrow$	3-6 times a week	"About once a month"	$\longrightarrow$	1 time a month or less
"1 to 3 times a week"	$\longrightarrow$	1-2 times a week	"Once or twice"	$\longrightarrow$	1-2 times a year

2. Appendix II: Equilibria for Θ_j population draws

Explicit calculation yields the following regions where equilibria occur. One may show that the I and IV equilibria occur for any collection of types[1]. However, for some draws of Θ_j the II and III equilibria do not occur, which is precisely when the corresponding sets below are empty. Given a draw $\Theta = \{\theta_i\}$ we order the θ_i in increasing order, and then the regions where each equilibrium occurs is as follows:

Table 2. Location of equilibria.

Equilibrium	$(1-p)^2 \in$
I	$(\frac{\theta_2}{\theta_4}, 1) \bigcap (\frac{\theta_3^2}{\theta_4^2}, 1)$
II	$(\frac{\theta_1}{\theta_4}, 1) \bigcap (\frac{\theta_2^2}{\theta_3^2}, 1) \bigcap (0, \frac{\theta_3^2}{\theta_4^2})$
III	$(\frac{\theta_1}{\theta_4}, 1) \bigcap (0, \frac{\theta_2}{\theta_4}) \bigcap (0, \frac{\theta_2^2}{\theta_3^2})$
IV	$(0, \frac{\theta_1}{\theta_4})$

3. Appendix III: Specific Model Examples

Sometimes the best way to communicate is with examples and applications. Here we include one of each for the model presented above.

Here we illustrate the case of $\theta_i = 5 - i,\ N = 4, k = 1$ under RE presented graphically above. When $p = 1$ or 0 many choices become uninteresting, so we tacitly ignore that case in what follows. From Proposition 1, for any p agent 1 always has $A_1^* = (\theta_2)$. Also from Proposition 1, since agent 2 only optimizes conditional on being unmatched when $t = 2$, it must be in that case that agent 1 is unmatched so for all p, $A_2^* = (\theta_1)$. Yet again from proposition 1 we need only consider each agent's optimal strategy in order of when their contacts are evaluated. Accordingly we have:

Agent 3: Now the probabilities of agent 1 and 2 being unmatched at $t = 3$ are the same so as $\theta_1 > \theta_2$, $\theta_1 \succ \theta_2$ for agent 3. As for choosing θ_4 we have:

$$\theta_1 \succeq \theta_4 \iff 4p(1 - p - (1-p)p) \geq p \iff p^2 - 2p + \frac{3}{4} \geq 0 \iff p \leq \frac{1}{2}$$

Agent 4: Since agent 4's optimal decision depends on Agent 3's action, we break this into two cases.

Case 1, $p \geq \frac{1}{2}$ and $A_3^* = (\theta_4)$: Again $\theta_1 \succ \theta_2$ so we have

$$\theta_1 \succeq \theta_3 \iff 4p(1 - p - (1-p)p) \geq 2p \iff p^2 - 2p + \frac{1}{2} \geq 0 \iff p \leq 1 - \frac{\sqrt{2}}{2}$$

Since $1 - \frac{\sqrt{2}}{2} < \frac{1}{2}$, θ_1 is never optimal for agent 4 in this case and $A_4^* = (\theta_3)$.

[1]In fact, with in any game with an even number of agents.

Case 2, $p \leq \frac{1}{2}$ and $A_3^* = (\theta_1)$: We have, where U indicates expected utilities

$$U(\theta_1) = 4p(1-p(2-p))(1-p)\ U(\theta_2) = 3p(1-p(2-p))\ U(\theta_3) = 2p(1-p)$$

So that after some algebra

$$\theta_1 \succeq \theta_3 \iff p \leq 1 - \frac{\sqrt{2}}{2}$$

$$\theta_1 \succeq \theta_2 \iff p \leq \frac{1}{4}$$

$$\theta_2 \succeq \theta_3 \iff p \leq \frac{1}{3} \text{ or } p = 1$$

Combinations of the relevant conditions then yields

$$A_4^* = \begin{cases} \theta_3, & \frac{1}{2} \geq p \geq \frac{1}{3} \\ \theta_2, & \frac{1}{3} \geq p \geq \frac{1}{4} \\ \theta_1, & \frac{1}{4} \geq p \geq 0 \end{cases}$$

Casual observation supports the following story:

Picture a bar filled with 50+ year-olds who are happily contacting each other and going home with one another. In walks a handsome, buff 25 year-old who everyone takes notice of. The 50+ year-olds abandon their earlier plans of contacting each other and overwhelmingly decide to chase the 25 year-old, the end result being that many 50+ year-olds go home alone and frustrated.

In other words, the high premium attached to some individuals creates a "tragedy of the sexual commons" where not only does individual behavior lead to an inefficient group outcome, but better options may in fact decrease expected utility for some agents.

Example. Better options can hurt low types.

Consider a market populated by N agents of which $\frac{N}{2}$ are of type θ_L and $\frac{N}{2}$ are of type $\theta_H > \theta_L$. We may construct similar examples for all $p \in (0,1)$ so we arbitrarily choose $p = \frac{1}{2}$ and since decisions are invariant to scaling types by a constant we normalize θ_L to 1. It is not hard to show that as long as $\theta_H < 4$ low types all contact one another in any equilibrium, and that for $\theta_H > 4$ every low type contacts a high type in any equilibrium. The expected utility of a low type in equilibrium, given θ_H is:

$$u_L(\theta_H) = \begin{cases} \frac{3}{4}, & \theta_H < 4 \\ \frac{\theta_H}{8}, & \theta_H > 4 \end{cases}$$

So for low types, utility is constant until $\theta_H > 4$ at which point a "frenzy of chasing" occurs in which no low type will ever go home with another low type. Low types will actually do better when $\theta_H \in (1,4)$ than when $\theta_H \in (4,6)$. In the language above, as θ_H crosses 4, the game transitions (violently) from a "sorting" equilibrium to a "chasing" equilibrium.

We add that this chasing comes to the benefit of high types, as illustrated by their expected utility jump when θ_H crosses 4 and high types match with low types as a "last resort":

$$u_H(\theta_H) = \begin{cases} \frac{3}{4}\theta_H, & \theta_H < 4 \\ \frac{3}{4}\theta_H + \frac{1}{8}, & \theta_H > 4 \end{cases}$$

4. Appendix IV: Additional Results and Applications

We present two measures of efficiency is this setting.

The first efficiency measure: compute the expected number of matches that will occur in a given RE equilibrium $\mathcal{E} = \{A_i^*\}$. Since outcomes are probabilistic, efficiency measures should be *ex ante* and an appropriate measure is the aggregate match function AM defined by:

$$\mathrm{AM}(\mathcal{E}) = 2 \sum_{H \in \mathcal{H}} \sum_{i \geq 1} \sum_{k > i} f(H | A_1^*, A_{-1}^*) \chi_H((i,k))$$

We are interested in

$$\mathrm{AM}^*(N,p) := \max_{\mathcal{E} \in \{\{A_i\}_{1 \leq i \leq N}\}} \mathrm{AM}(\mathcal{E})$$

The AM^* measure can be directly compared to the case of homogeneous types.

The second efficiency measure: compute the expected aggregate utility. The (expected) aggregate utility function AU for a RE equilibrium $\mathcal{E} = \{A_i^*\}$ is defined by:

$$\mathrm{AU}(\mathcal{E}) = \sum_{H \in \mathcal{H}} \sum_{i \geq 1} \sum_{k > i} [\theta_i + \theta_k] f(H | A_1^*, A_{-1}^*) \chi_H((i,k))$$

and again maximum efficiency can be defined as

$$\mathrm{AU}^*(\Theta,p) := \max_{\mathcal{E} \in \{\{A_i\}_{1 \leq i \leq |\Theta|}\}} \mathrm{AU}(\mathcal{E})$$

We note that:

(1) In the case of homogeneous types, the two measures coincide.

(2) Both efficiency measures increase in p.

(3) $\frac{\mathrm{AM}^*(N,p)}{N}$ and $\frac{\mathrm{AU}^*(\Theta,p)}{N}$ should be something like weakly increasing in N from comparison of the domains for N and $2N$.

The first efficiency measure has a direct interpretation, that of *loneliness* or the expected percentage of agents who go home alone, which we define as

$$L(\mathcal{E}) = \frac{N - \mathrm{AM}^*(\mathcal{E})}{N}$$

For simplicity we will often use the following assumptions:

Condition 4.1. $k = 1$, $p \in (0,1)$, $N > 2$ is even, and beliefs satisfy RE. Also, Condition 1 holds.

Under Condition 1, Proposition 1 has the interpretation that only the actions of higher types matter for each agent's optimal decision. Condition 2 adds the interpretation that agents are upward looking, in the sense they their optimal decision will always be to contact an agent who is one type less or better. We state this as:

Proposition 4.2. *[Upward Looking Agents, or No Love Triangles] Assume Condition 2. In any equilibrium* $\mathcal{E} = \{A_i^* = (\theta_{j(i)})\}$, *necessarily* $j(i) \geq i - 1$.

Proof. Easily follows from Proposition 1 and consideration of each agent's decision problem. □

We remark that this rules out "love triangles," which to a large extent is the idea which motivates Gale and Shapley's stable marriage matching equilibrium. Condition 2 also allows for the relatively compact efficiency calculations:

Proposition 4.3. *Assume Condition 2. Let* $\mathcal{E}_s$ *be a sorting equilibrium and* $\mathcal{E}_c$ *be a chasing equilibrium. Then*

(1) $\mathrm{AM}(\mathcal{E}_s) = Np(2-p)$ so $L(\mathcal{E}_s) = (1-p)^2$

(2) $\mathrm{AU}(\mathcal{E}_s) = N\overline{\theta}p(2-p)$

(3) $\mathrm{AM}(\mathcal{E}_c) = 2[1-(1-p)^N]$

(4) $\mathrm{AU}(\mathcal{E}_c) > [1-(1-p)^N]\theta_1$

Proof. Direct calculation.

□

Example. Assume Condition 2. Then:

(1) Homogeneous θ results in a sorting equilibrium up to ties.
(2) For every p and N and θ there exists some $\lambda > 0$ such that for $\Theta_\lambda := \{\theta_i : \ \theta_i = \theta + (N-i)\lambda\}$ the unique equilibrium is a sorting equilibrium. In particular $\lambda \leq \frac{p(2-p)\theta}{N}$ is sufficient.
(3) Heterogeneous θ are not necessarily efficient in the aggregate matching sense.

Proof. Sketches can be found in the appendix.

□

Example. Assume Condition 2. Sufficiently large θ_1 results in a chasing equilibrium.

Proposition 4.4. *Under condition 2, sorting equilibria are efficient in the aggregate matches, and equilibria not consisting of 2-person cliques are inefficient in aggregate matches.*

Proof. A sketch of a proof is in the appendix.

As one would expect, when all agents are "trying their luck" with the highest type in the game, the total number of matches is minimized.

□

Proposition 4.5. *Under condition 2, chasing equilibria minimize aggregate matches.*

Proof. A sketch of a proof is in the appendix.

□

This result shows that a sorting equilibrium, or rather, "cliquing" does in fact maximize aggregate matches in all cases. However, no such result holds for aggregate utility: since $\mathrm{AU}(\mathcal{E}_c) > [1-(1-p)^N]\theta_1$, for sufficiently small p and large $(\theta_1 - \overline{\theta})$ we have $\mathrm{AU}(\mathcal{E}_c) > \mathrm{AU}(\mathcal{E}_s)$.

5. Appendix V: Formal Statements and Proofs

Algorithm 1 Match evaluation procedure.

Note that we have reversed the order of θ_i from that in the main text for ease of notation.

(1) Fix $p \in (0,1]$ and let $M_1 = \phi$, $E_1 = \phi$, $t = 1$.

(2) Find the least i, say I, that is unmatched and unexhausted, namely $I = \min_{i \notin M_t \bigcup E_t} i$.

(3) (For bookkeeping.) For s, $t \leq s \leq I$ let $M_s = M_t$ and $E_s = E_t$. Let $t = I$.

(4) Let $\tilde{A_I}$ be the vector of I's contacts who are still unmatched, preserving the original order.

(5) While I is unmatched, I "goes through" her contacts a_k in order and is "semi-matched" to each with probability p.

(6) If during the process I is semi-matched to say, j, then if $j \in E_t$, let $M_{t+1} = M_t \bigcup \{I, j\}$, $E_{t+1} = E_t \backslash \{j\}$, increment t and go to Step 1; otherwise if $j \notin E_t$ let $M_{t+1} = M_t \bigcup \{I\}$, $E_{t+1} = E_t$, increment t and go to Step 1

(7) If I is never semi-matched after exhausting her contacts, let $M_{t+1} = M_t$, $E_{t+1} = E_t \bigcup \{I\}$, increment t and go to Step 1.

Proposition. *Suppose μ and μ' are two possible beliefs for agent i and that*

$$B(j|\mu) = B(j|\mu') \; \forall j < i$$

then

$$\arg\max_A U_i(A, \mu) = \arg\max_A U_i(A, \mu')$$

In other words, agent i's optimal decision depends only on her beliefs of agents who evaluate matches before i.

Proof. We wish to show that

$$\arg\max_A \sum_{H \in \mathcal{H}} \sum_k u_i(\theta_k) f(H|A, \mu) \chi_H((i,k)) = \arg\max_A \sum_{H \in \mathcal{H}} \sum_k u_i(\theta_k) f(H|A, \mu') \chi_H((i,k))$$

Denote by S_i the event that in the clearing process above that i is unmatched in Step 1 when $t = i$. Agent i can only affect her probability of being matched to other agents when $t = i$, and so is interested only in the induced probability distribution of unmatched agents at $t = i$ given S_i has occurred, namely $f(H|A, \mu, S_i)$. For any beliefs

μ_i and contact choice A_i we have

$$f(H|A_i,\mu) = f(H|A_i,\mu_i,S_i)\Pr(S_i|A_i,\mu_i) + f(H|A_i,\mu_i,\neg S_i)[1-\Pr(S_i|A_i,\mu_i)]$$

A_i can affect matching only if S_i occurs hence:

$$f(H|A_i,\mu) = f(H|A_i,\mu_i,S_i)\Pr(S_i|\mu_i) + f(H|\mu_i,\neg S_i)[1-\Pr(S_i|\mu_i)]$$

Since $f(H|\mu_i,\neg S_i)[1-p(S_i|\mu_i)]$ is unaffected by A_i we may ignore it in the maximization problem, as well as the multiplicative constant $\Pr(S_i|\mu_i)$. Thus we have shown that

$$\text{(P1)}\ \max_A \sum_{H\in\mathcal{H}}\sum_k u_i(\theta_k)f(H|A,\mu_i)\chi_H((i,k))$$

is equivalent to

$$\text{(P2)}\ \max_A \sum_{H\in\mathcal{H}}\sum_k u_i(\theta_k)f(H|A,\mu_i,S_i)\chi_H((i,k))$$

Let $M_{a,b}$ denote the event that agent a is matched to agent b at $t=i$, and let $P(M)$ denote the space of all combinations of $M_{a,b}$. In evaluating $\Pr(m\in P(M))$, since matches can only have occurred before $t=i$, only the first $i-1$ values of $B(\cdot|\mu_i)$ matter. Also each $m\in P(M)$ is independent of A_i so

$$f(H|A,\mu_i,S_i) = \sum_{m\in P(M)} f(H|A,\mu_i,S_i,m)\Pr(m|\mu_i,S_i)$$

By assumption $B(j|\mu)=B(j|\mu')\ \forall j<i$ and therefore

$$\Pr(S_i|\mu) = \Pr(S_i|B(\cdot|\mu)) = \Pr(S_i|B(\cdot|\mu')) = \Pr(S_i|\mu')$$

$$\Pr(m|\mu) = \Pr(m|B(\cdot|\mu)) = \Pr(m|B(\cdot|\mu')) = \Pr(m|\mu')$$

Using both, since

$$\Pr(m|\mu,S_i) = \frac{\Pr(m|\mu)-\Pr(m|\mu,\neg S_i)[1-\Pr(S_i|\mu)]}{\Pr(S_i|\mu)}$$

For each m the joint events (m,S_i) and $(m,\neg S_i)$ are mutually exclusive and exhaustive so when S_i precludes m certainly

$$\Pr(m|\mu,S_i) = \Pr(m|\mu',S_i) = 0$$

and otherwise $\Pr(m|\mu,\neg S_i) = \Pr(m|\mu',\neg S_i) = 0$ so that

$$\Pr(m|\mu,S_i) = \frac{\Pr(m|\mu)-\Pr(m|\mu,\neg S_i)[1-\Pr(S_i|\mu)]}{\Pr(S_i|\mu)} =$$

$$\frac{\Pr(m|\mu)}{\Pr(S_i|\mu)} = \frac{\Pr(m|\mu')}{\Pr(S_i|\mu')} = \Pr(m|\mu',S_i)$$

Thus (P2) is equivalent to

$$\text{(P3)}\ \max_A \sum_{H\in\mathcal{H}} \sum_k \sum_{m\in P(M)} \alpha_{k,i} f(H|A,\mu_i,S_i,m)$$

with

$$\alpha_{k,i} := u_i(\theta_k)\Pr(m|\mu,S_i)\chi_H((i,k)) = u_i(\theta_k)\Pr(m|\mu',S_i)\chi_H((i,k))$$

for each k.

Finally, denote by S_i' the event that in the clearing process above that i is unmatched in Step 1 when $t = i+1$. From the clearing process, one can see that $\Pr(S_i'|S_i,\cdot)$ is constant, so

$$f(H|A,\mu_i,S_i,m) =$$

$$f(H|A,\mu_i,S_i,m,S_i')\Pr(S_i'|S_i,\cdot) + f(H|A,\mu_i,S_i,m,\neg S_i')[1-\Pr(S_i'|S_i,\cdot)]$$

Conditional on S_i', all A_i yield the same probability, so $f(H|A,\mu_i,S_i,m,S_i')\Pr(S_i'|S_i,\cdot)$ drops out of the maximization problem. Conditional on $\neg S_i'$, no clearing that occurs after $t = i$ can affect $p_i((i,k)|A_i,S_i,m,\mu_i,\neg S_i')$ hence

$$f(H|A,B(\cdot|\mu),S_i,m,\neg S_i') = f(H|A,B(\cdot|\mu'),S_i,m,\neg S_i')$$

and again $1-\Pr(S_i'|S_i,\cdot)$ drops out. All together, we have that (P3) is equivalent to

$$\text{(P4)}\ \max_A \sum_{H\in\mathcal{H}} \sum_k \sum_{m\in P(M)} \alpha_{k,i} f(H|A,\mu_i,S_i,m,\neg S_i')$$

therefore A^* solves (P1) for $\mu_i = \mu$ iff A^* solves (P4) with $\mu_i = \mu,\mu'$ iff A^* solves (P1) with $\mu_i = \mu'$.

□

Proposition. *A RE equilibrium always exists.*

Proof. Proposition 1 shows that μ_1 is irrelevant to Agent 1's problem, and since $\Pr(S_1) = 1$ the problem simplifies to (from (P4))

$$\max_A \sum_k u(k) p_i((i,k)|A, m = \text{"all single"}, \neg S_1')$$

$$\max_A \sum_{H\in\mathcal{H}} \sum_k u_1(\theta_k) f(H|A, m = \text{"all single"}, \neg S_1')\chi_H((i,k))$$

the solution to which is easily seen to be for Agent 1 to contact the $2,\ldots,k+1$ agents since θ_j is decreasing in j. Fix an optimal strategy for Agent 1, A_1^* and any beliefs μ_1 (all are consistent with A_1^*). Proposition 1 allows us to construct and equilibrium by induction: Suppose Agents $1,\ldots,n$ choose optimal strategies $\{A_i^*\}_{i\le n}$ with RE beliefs for $i \leq n$, say $\{\mu_i\}_{i \leq n}$. Let μ'_{n+1} be a RE belief for $\{A_i^*\}_{i \leq n}$ and let A_{n+1}^* be optimal for μ'_{n+1}. Choose new beliefs $\{\mu'_i\}_{i \leq n}$ to be RE for $\{A_i^*\}_{i \leq n+1}$ and by proposition 1 each A_i^* is still optimal for $\{\mu'_i\}_{i \leq n+1}$.

□

Proposition. *Let $k = 1$, and for any beliefs $\{\mu_i\}$ $\exists \epsilon > 0$ s.t. in all equilibria,*

(1) *If $p \in [0, \epsilon)$, $A_i^* = (\theta_1)$ $\forall i > 1$.*
(2) *If $p \in (1-\epsilon, 1]$ and beliefs satisfy RE, $A_i^* = (\theta_{\rho(i)})$.*

Proof. (1, sketch.) Let S_i denote the event that agent i is unmatched when $t = i$. We have for each i, where μ is taken over possible beliefs for agent i, that:

$$\alpha_i(p) := \min_{\mu} \Pr(S_1 | S_i, \mu) = (1-p)^{i-1}$$

so that

$$\alpha(p) := \min_i \alpha_i = (1-p)^{N-1}$$

Since $\alpha(p) \longrightarrow 1$ as $p \longrightarrow 0$ and $u(\theta_1) > u(\theta_2)$, $\exists \epsilon > 0$ s.t. $\forall p \in [0, \epsilon)$ we have

$$u(\theta_1)\alpha(p) > u(\theta_2) \geq u(\theta_j) \ \forall j > 1$$

One can then (tediously) show that $\forall p \in [0, \epsilon)$ and $i > 1$ that $A_i^* = (\theta_1)$.

(2, sketch.) It is easy to see that for $i \in \{1, 2\}$ that $A_i^* = (\theta_{\rho(i)})$. Fix $m < n$ and suppose for $i \in \{1, \ldots, 2m\}$ that $A_i^* = (\theta_{\rho(i)})$. We claim $\exists \epsilon_m > 0$ s.t. for $i \in \{2m+1, 2m+2\}$ and $p \in (1-\epsilon_m, 1]$ we have $A_i^* = (\theta_{\rho(i)})$ (this uses Proposition 1 and thus RE). Construct ϵ_m for all $m < n$ and let

$$\epsilon = \min_m \epsilon_m$$

Finally, we note that in both cases the value of ϵ depends not only on N but also on $\{\theta_i\}$.

□

Proposition. *Under condition 1, sorting equilibria are efficient in the aggregate matches, and equilibria not consisting of 2-person cliques are inefficient in aggregate matches.*

Proof. (Sketch) Take any contact network which does not correspond to a sorting equilibrium. Look at disconnected subgraphs $\{S_j\}$ and start with the lowest numbered vertex, say v_1 in a subgraph, say S. Let $R(v_1)$ be the vertex reachable from v_1 and create a new graph G_1 consisting of v_1 and $R(v_1)$, as well as $(v_1, R(v_1))$. Now find v_2 where v_2 is the least numbered vertex s.t. $v_2, R(v_2) \notin V(G_1)$. Proceed inductively, terminating at v_m where necessarily $m \leq \frac{|S|}{2}$. Edges in G_m give expected matches of no more than $2p$. Edges coming from the $R(v_i)$ in G_m give expected matches of no more than $2p(1-p)$. Edges corresponding to vertexes not in G_m, say v_a must connect to lowered numbered vertexes or vertexes with incoming edges that originate at vertexes with indexes lower that a, hence give expected matches of no more than $2p(1-p)$. Hence if any v_a exists the contact network is inefficient in aggregate matches. If no v_a exists and $|S_j| = 2\ \forall j$ this gives the same AM as a sorting equilibrium. If there is an S_j with $|S_j| > 2$ then constructing G_m above shows that some edge from a $R(v_i)$ connects two $\{v_b, R(v_b)\}$ pairs which can be shown to be inefficient. Thus a necessary condition for AM efficiency is 2-person cliques which in turn all give the same AM. Since sorting equilibria yield two person cliques, they are efficient in AM.

□

Proposition. *Under condition 2, chasing equilibria minimize aggregate matches.*

Proof. (Sketch) Let $\mathrm{AM}_E(p)$ denote aggregate matches under eq'm E at p. Fix p and let C denote the chasing eq'm and

□

Example. Assume Condition 2. Then:

(1) Homogeneous θ results in a sorting equilibrium up to ties.
(2) For every p and N and θ there exists some $\lambda > 0$ such that for $\Theta_\lambda := \{\theta_i :\ \theta_i = \theta + (N-i)\lambda\}$ the unique equilibrium is a sorting equilibrium. In particular $\lambda \leq \frac{p(2-p)\theta}{N}$ is sufficient. Similarly, if $\frac{\theta_1 - \theta_N}{\theta_N} < p(2-p)$ the unique equilibrium is sorting.
(3) Heterogeneous θ are not necessarily efficient in the aggregate matching sense.

Proof. (1) (Sketch) Consider the contact network consisting of all vertexes and the edges originating from the first $\frac{N}{2}$ vertexes. Using proposition 1, we claim that in any equilibrium this network is disconnected. Thus each agent i, $i > \frac{N}{2}$ has an incoming edge from an agent $j(i)$ with $j(i) \leq \frac{N}{2}$. It is then easy to see that i's unique optimal action is to contact $j(i)$, which results in a equilbrium consisting of 2 person cliques. In fact, the first $\frac{N}{2}$ contacts can be chosen so that this is a sorting equilibrium. Interestingly, small perturbations to type, negatively correlated with i also will induce a sorting equilibrium.

(2) (Sketch) Appealing to Proposition 1, we proceed inductively. Now assume for m odd assume there exists an $\epsilon_{m-1} > 0$ s.t. for $\Theta = \Theta_{\epsilon_{m-1}}$ and $\forall n < m$ we have $A_n^* = (\theta_{\rho(n)})$. Under our assumptions and the inductive hypothesis, direct comparison shows that if $A_m^* = (\rho(m))$ then $A_{m+1}^* = (\rho(m+1))$. Fix ϵ, and consider the following:

Case 1: $j < m+1$

$$\theta_j \succeq \theta_{m+1} \implies p(\theta + (N-j)\epsilon) - p(\theta + (N-m-1)\epsilon) \geq 0 \iff m+1-j \geq 0$$

which never holds, so in this case $\theta_j \prec \theta_{m+1}$.

Case 2: $j > m$

$$\theta_j \succeq \theta_{m+1} \implies p[1-p(2-p)](\theta + (N-j)\epsilon) - p(\theta + (N-m-1)\epsilon) \geq 0 \iff$$

$$g(p) := \{[1-p(2-p)](N-j) - (N-m-1)\}\epsilon - p(2-p)\theta \geq 0$$

Observe that

$$g(p) < N\epsilon - p(2-p)\theta$$

so $\exists \epsilon > 0$ s.t. $\forall m$ as above and $j \neq m+1$ that $\theta_j \prec \theta_{m+1}$.

For $m = 1$, clearly $A_1^* = (\theta_2)$ and taking that into consideration, $A_2^* = (\theta_1)$. Let $\epsilon_0 = \epsilon$ and induction will follow with $\lambda = \epsilon$. We note that $\lambda \leq \frac{p(2-p)\theta}{N}$ is sufficient for the result.

(3) Direct comparison using the preceding proposition shows that $\text{AM}(\mathcal{E}_s) > \text{AM}(\mathcal{E}_c)$ $\forall p \in (0,1)$. By above $\exists p$ s.t. $\mathcal{E} = \{A_i^*\}$ is a chasing equilibrium, which must therefore be inefficient in aggregate matches.

$\square$